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CP and CPT Symmetry Violations, Entropy and the Expanding Universe

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Abstract

The conjecture linking together statistical and cosmological time arrows implies interpreting the contracting phase in an oscillating cosmological model as a time-inverted expansion. The Fitch-Cronin effect now requires a redefinition of which is matter and which is antimatter in this time-inverted picture. However, if *CPT* is also broken, then the *T*-violating effects yield a different set of laws altogether for such reactions.

1. *Microscopic Irreversibility*

It is by now a well-established fact that the Fitch-Cronin effect (Christenson, *et al.*, 1964) represents a violation of the combined *CP* symmetry of the known interactions at the microscopic level. Strong and electromagnetic interactions respect both *C* (generalized charge parity) and *P* (space parity). Weak interactions break *C* and *P* but do respect the product operation *CP*. Using available experimental data one can deduce that the new effect may originate in either a *P*-conserving *C*-violating new 'milli-strong' interaction (of strength similar to electromagnetic but uncoupled to photons), or in a 'milli-weak' small component of the weak interactions, or alternatively in a 'super-weak' new force of order $10^{-9}\ G_{\text{Fermi}}$.

It would seem at first sight that one might salvage *T* invariance, i.e. symmetry under microscopic time-reversal, by abandoning *CPT* invariance. However, the experimental situation is such that *T* is found to be violated anyhow (Casella, 1968, 1969; Achiman, 1969). Whether or not *CPT* holds only affects the size of this violation. With past experience having gradually forced us to abandon first *P*, then *CP*, we shall consider here both *CPT*-invariant and *CPT*-non-invariant situations.

Two recent studies (Zweig, 1967; Ne'eman, 1969) have independently dwelt upon problems arising from the possible links between the new microscopic 'arrow of time' and the statistical and cosmological ones. In a general way, the new effect brings about an additional irreversibility, on

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top of the statistical one. It is difficult to conceive of situations in which thermodynamical irreversibility can be made to vanish, so as to bring out the direct effects of microscopic irreversibility. However, we would like to show in this paper how the usual assumption about the interrelations between statistical and cosmological time-arrows brings about just such a situation.

A common impression† among astrophysicists and cosmologists has been that the two arrows are linked together. An expanding universe is then the only conceivable one; roughly, in a contracting universe entropy would have to decrease, the universe changing from a disordered spread-out (large phase-space) state to a concentrated, ordered (minute phase-space) state. This is, of course, an oversimplified statement, since models can be constructed in which an oscillating universe would go from one concentrated state to the next one with a permanent increase of entropy, for example. The simplicity of the first view has however led to its adoption as a plausible conjecture, fitting in nicely with the ideas of steady-state theory, and adaptable to other models.

Consider now an oscillating model. The above view implies a reversal of phase-space considerations in the contracting phase, so as to make it appear as an expansion, from the statistics. Suppose the only available matter in the universe were a beam of K^0 (or $\bar{K}^0$) mesons. These would decay into π mesons; the fractional number of K^0 and $\bar{K}^0$ mesons remaining in a K^0 (or $\bar{K}^0$) beam is at time t [e.g. Rosen's preferred frame (Rosen, 1968)]

$$\begin{aligned} R^{\pm}(t) = \{\tfrac{1}{2}|1-\epsilon^2+\delta^2|^{-2}\}\{(1+|\epsilon|^2+|\delta|^2+2\operatorname{Re}\epsilon\delta^*)|1\mp\epsilon\pm\delta|^2 \times \\ \times \exp(-\gamma_S t) + (1+|\epsilon|^2+|\delta|^2-2\operatorname{Re}\epsilon\delta^*)|1\mp\epsilon\mp\delta|^2 \times \\ \times \exp(-\gamma_L t) \pm (2\operatorname{Re}\epsilon + 2i\operatorname{Im}\delta)(1\mp\epsilon^*\pm\delta^*)(1\mp\epsilon\mp\delta) \times \\ \times \exp[-\tfrac{1}{2}(\gamma_S+\gamma_L)t + i\Delta m t] \pm (2\operatorname{Re}\epsilon + 2i\operatorname{Im}\delta) \times \\ \times (1\mp\epsilon\pm\delta)(1\mp\epsilon^*\mp\delta^*)\exp[-\tfrac{1}{2}(\gamma_S+\gamma_L)t - i\Delta m t]\} \end{aligned}$$

where ϵ is the *CP* violation parameter, δ the *CPT* violation. Both are complex numbers with the upper sign for K^0 and the lower one for $\bar{K}^0$. To first order in ϵ and δ (both are experimentally determined to be small) we may write,

$$\begin{aligned} R^{\pm}(t) = \{\tfrac{1}{2} \mp \operatorname{Re}(\epsilon-\delta)\exp(-\gamma_S t) + \{\tfrac{1}{2} \mp \operatorname{Re}(\epsilon+\delta)\}\exp(-\gamma_L t) \\ \pm \{\operatorname{Re}\epsilon - i\operatorname{Im}\delta\}\exp[-\tfrac{1}{2}(\gamma_S+\gamma_L)t + i\Delta m t] \\ \pm \{\operatorname{Re}\epsilon + i\operatorname{Im}\delta\}\exp[-\tfrac{1}{2}(\gamma_S+\gamma_L)t - i\Delta m t]\} \end{aligned} \tag{1.1}$$

† See for example T. Gold's article in *Recent Developments in General Relativity*, p. 225. Pergamon–Macmillan, New York and Warsaw, 1962; Gell-Mann, M. (1967) comments in *Proceedings of the Temple University Panel on Elementary Particles and Relativistic Astrophysics*; Salpeter, E. (1968). Lecture at the Tel-Aviv Seminar on Astrophysics.

2. *The CPT Conserving Case*

Suppose we now go over to a time $\tau - t$, with τ the oscillation period of the universe; or in fact to $-t$, which should look the same as $\tau - t$. Our pions are now being squeezed back to remake the original K^0 or $\bar{K}^0$ beam. Take the case where this was a K^0 beam; to describe this production process, we have to time-invert our formula. If *CPT* is conserved, a theorem (Lee, *et al.*, 1957) states that the time-reversed amplitude is the same as the *CP*-inverted one, to first order in the *CP* violation:

$$\begin{aligned}\langle B|H_{\text{viol}}|A\rangle^* &\equiv \langle TB|TH_{\text{viol}}T^{-1}|TA\rangle = \langle TB|C^{-1}P^{-1}H_{\text{viol}}PC|TA\rangle \\ &= \langle \bar{B}|H^+_{\text{viol}}|\bar{A}\rangle - \langle \bar{B}|H^-_{\text{viol}}|\bar{A}\rangle\end{aligned}$$

for A, B spinless states, and H^+_{viol}, H^-_{viol} denoting the even and odd parity parts of H_{viol}. The two amplitudes are orthogonal to this order, so that

$$|\langle TB|TH_{\text{viol}}T^{-1}|TA\rangle|^2 = |\langle \bar{B}|H_{\text{viol}}|\bar{A}\rangle|^2 \tag{2.1}$$

The implication is that we have to use the $\bar{K}^0$ decay formula to describe K^0 production at $-t$. If we now consider the contraction phase between $\tau/2$ and τ as an expansion from the concentrated state at τ, we invert $t \to -t$ in the decay formula. This will then turn the production of K^0 into the decay of $\bar{K}^0$:

$$P^+(-t) = R^-(t) \tag{2.2}$$

Cosmologically, we learn that in the entropy-symmetric description, *a contracting matter universe is the same as an expanding antimatter universe.* Both descriptions use the same laws of nature, provided we replace matter by antimatter and vice versa. We note that this is now no trivial change, since the two do behave differently. Within one universe, they can now be distinguished in the relative sense. For instance,

$$K_L{}^0 \to \pi^{\mp} + e^{\pm} + \overset{(-)}{\nu}_e \qquad \text{or} \qquad \to \pi^{\mp} + \mu^{\pm} + \overset{(-)}{\nu}_\mu$$

are *CP*-violating decays (Christenson, *et al.*, 1964), with the measured asymmetry ratio between rates

$$r = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \sim 2.10^{-3}$$

(the sign in $\Gamma_{+,-}$ corresponds to the charge of the resulting electron or muon).

Communicating with physicists in a distant galaxy, we simply ask them to make K^0 or $\bar{K}^0$ mesons, to use the long-lived component $K_L{}^0$ and observe the asymmetry. We can tell them that when r is positive, the more numerous leptons are positrons and should be considered as antimatter by our conventions.

Returning to the cosmological situation we note that with *CPT* invariance, all textbooks will look the same in the time-inverted expanding universe except for a matter–antimatter replacement. What if *CPT* itself fails?

3. *The CPT-Violating Case*

The Lee-Oehme-Yang theorem (Lee, *et al.*, 1957) (2.1) does not hold. To explore this situation we shall study the behavior of ϵ and δ, the two complex parameters in (1.1) under the two operations we used:

(a) a matter–antimatter replacement (i.e. *CP*);
(b) overall time inversion (actually *T* since we do neutralize the effects of phase-space in the cosmological picture).

Under *CP*, both ϵ and δ change sign. In the two-dimensional $\psi(K^0, \bar{K})$ space, with

$$i\frac{d}{dt}\psi = (M - i\Gamma)\psi = \Lambda\psi \tag{3.1}$$

where M and Γ are 2×2 hermitean matrices, Λ a general 2×2 one, $\lambda_{S,L}$ the two eigenvalues (widths and masses) of Λ, short- and long-lived, respectively

$$\epsilon \simeq \frac{\Lambda_{12} - \Lambda_{21}}{\lambda_S - \lambda_L} \tag{3.2}$$

Using the Wigner-Weisskopf method,

$$\begin{aligned}\Lambda_{12} &\simeq \langle K^0|H_W|\bar{K}^0\rangle + \sum_f \frac{1}{M_{K^0} - M_f + i\epsilon}\langle K^0|H_W|f\rangle\langle f|H_W|\bar{K}^0\rangle \\ \Lambda_{21} &\simeq \langle \bar{K}^0|H_W|K^0\rangle + \sum_f \frac{1}{M_{\bar{K}^0} - M_f + i\epsilon}\langle \bar{K}^0|H_W|f\rangle\langle f|H_W|K^0\rangle\end{aligned} \tag{3.3}$$

Since *CP* exchanges K^0 and $\bar{K}^0$, $\Lambda_{12} \leftrightarrow \Lambda_{21}$ and thus the *CP*-violation parameter changes sign,

$$\epsilon \to -\epsilon \tag{3.4}$$

For the *CPT*-violation parameter ($\delta = 0$ if *CPT* is conserved) we have

$$\delta \simeq \frac{\Lambda_{11} - \Lambda_{22}}{\lambda_S - \lambda_L} \tag{3.5}$$

$$\begin{aligned}\Lambda_{11} &\simeq M_{K^0} + \langle K^0|H_W|K^0\rangle + \sum_f \frac{1}{M_{K^0} - M_f + i\epsilon}\langle K^0|H_W|f\rangle\langle f|H_W|K^0\rangle \\ \Lambda_{22} &\simeq M_{\bar{K}^0} + \langle \bar{K}^0|H_W|\bar{K}^0\rangle + \sum_f \frac{1}{M_{\bar{K}^0} - M_f + i\epsilon}\langle \bar{K}^0|H_W|f\rangle\langle f|H_W|\bar{K}^0\rangle\end{aligned} \tag{3.6}$$

so that

$$\delta \to -\delta \tag{3.7}$$

We now try *T*. Wigner time-reversal acts so that

$$\psi(x, t) \to \psi^*(x, -t)$$

to preserve the Schrödinger equation (3.1)

$$i\frac{d}{dt}\psi_T{}^* = (M^* - i\Gamma^*)\,\psi_T{}^* \tag{3.8}$$

because $M_T = M$, but $\Gamma_T = -\Gamma$. Since both M and Γ are hermitean, $M_{12}^* = M_{21}$, $\Gamma_{12}^* = \Gamma_{21}$ and we see in (3.2) that

$$\epsilon_T = -\epsilon \tag{3.9}$$

but from (3.5)

$$\delta_T = \delta \tag{3.10}$$

This then answers our question. The new $P^+(-t)$ will look like $R^-(t)$ as far as the ϵ contribution is concerned, but will stay as in $R^+(t)$ for the δ terms. The most general case will then consist of *a Universe where replacing contraction by a time-inverted expansion implies that the resulting universe will have different laws of nature*! No matter–antimatter redefinition can settle this change.

The formulae (1.1) thus correspond to 'true' decays. In the time-inverted expanding universe they will become,

$$\begin{aligned} R_T{}^{\pm}(t) = {} & \{\tfrac{1}{2} \pm \mathrm{Re}\,(\epsilon + \delta)\}\exp(-\gamma_S t) + \{\tfrac{1}{2} \pm \mathrm{Re}\,(\epsilon - \delta)\}\exp(-\gamma_L t) \\ & \mp \{\mathrm{Re}\,\epsilon + i\,\mathrm{Im}\,\delta\}\exp[-\tfrac{1}{2}(\gamma_S + \gamma_L)\,t + i\Delta mt] \\ & \pm \{\mathrm{Re}\,\epsilon - i\,\mathrm{Im}\,\delta\}\exp[-\tfrac{1}{2}(\gamma_S + \gamma_L)\,t - i\Delta mt] \end{aligned} \tag{3.11}$$

The coefficients of the two diagonal states' decay curves have been inverted, besides changing from K^0 to $\bar{K}^0$. Another inversion occurs in the coefficients of the mixed oscillating terms. *The time-inverted expanding universe describes a sequence of states which does not appear in the history of the original contracting universe.*†

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† The only way of achieving universe-inversion symmetry would now consist in the introduction of a mirror-universe. The complete (double) universe would then reproduce the same set of states in either time direction.